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Renormalization group evolution in the nonlinear supersymmetric standard model

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Abstract. We consider the Higgs sector of a nonlinear supersymmetric standard model. Using the RGE and assuming supersymmetry to be broken at the Planck scale, radiative electroweak symmetry breaking is investigated. An upper bound of the mass of the lightest scalar Higgsboson is calculated.

1 Introduction

Supersymmetry (SUSY) is one of the most important theoretical discoveries since the advent of the nonabelian gauge theories. For almost twenty years the phenomenology of SUSY models has been studied. Almost all of these models are linear supersymmetric models, i.e., SUSY is realized linearly. Linear SUSY models require a SUSY partner to every conventional particle. Search for SUSY particles is one of the main goals of the present and future collider experiments. So far no SUSY partners have been found. However SUSY may as well be realized nonlinearly [1]. A characteristic property of the nonlinear realization is that no SUSY partners are required. In global nonlinear SUSY models the only additional field that has to be introduced is the Akulov-Volkov (A-V) field, a Goldstino. But in experiment no Goldstino has been observed [2]. A possibility to avoid the massless physical Goldstino is to go to curved space, to supergravity. The formalism for extending the standard model nonlinear supersymmetrically in curved space was developed by Samuel and Wess [3]. In supergravity the Goldstino can be gauged away; the massless Gravitino absorbs the Goldstino and becomes massive, whereas the Graviton remains massless. In the limit of flat space, where the supergravity multiplet decouples from the ordinary matter, the fermion particle spectrum is the same as in the standard model. The only reminiscence of SUSY manifests itself in the Higgs sector. The Higgs sector has to be extended as in the case of linear SUSY models. Recently a general nonlinear SUSY standard model was constructed [4].

This model contains two Higgs doublets and a Higgs singlet and is a nonlinear SUSY alternative to the linear SUSY model, the NMSSM. It was shown that there are typical differences in the structure of the Higgs potential between these two models. Physical consequences of this nonlinear supersymmetric standard model in the flat space limit and on tree level were investigated, in particular, how to test the model at future e^+e^--Colliders [5].

In this paper we consider the contributions of radiative correction. The point is that our model is a sugra model and thus the initial conditions for the parameters should be fixed at the Planck scale (M_P). For example the coupling of quartic Higgs term arising from the D-terms are related to the gauge couplings. In tree level these relations are assumed to hold also at the electroweak scale (M_E). In this paper we assume these relation hold at Planck scale and determine the evolution of the parameters using renormalization group (RG) equations. In Sect. 2 a brief introduction to the model is given. In Sect. 3 the RG equations of the model and initial condition at Planck scale are given. In Sect. 4 the upper bound of the mass of the lightest Higgs scalar is derived. In Sect. 5 the allowed parameter range yielding correct electroweak symmetry breaking is investigated.

2 The model

The complete Lagrangian in curved space is given by

$$\mathcal{L} = \mathcal{L}_{\text{gravity+av}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{gauge+matter}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{HPS}} \ . \quad (1)$$

$\mathcal{L}_{\text{gravity+av}}$ is the Lagrangian for the supergravity multiplet and the A-V field. It contains a constraint term which generate a negative contribution to the background vacuum energy density. With the contribution of the A-V field term to the vacuum energy density being positive they can be adjusted to cancel the cosmological constant. $\mathcal{L}_{\text{gauge}}$ is the pure gauge term of SU(3) × SU(2) × U(1). $\mathcal{L}_{\text{gauge+matter}}$ contains among other things terms which in the flat space limit reduce to the D-term of the Higgs po-

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tential. The last term $\mathcal{L}_{\mathrm{HSP}}$ reduces in flat space limit to the F-term of the Higgs potential.

The full Higgs potential in flat space limit is given by

$$\begin{aligned} V = &\frac{1}{8}(g_1^2+g_2^2)(|H_1|^2-|H_2|^2)^2+\frac{1}{2}g_2^2|H_1^+H_2|^2 \\ &+\mu_1^2|H_1|^2+\mu_2^2|H_2|^2+\mu_0^2|N|^2 \\ &+\tilde{\lambda}_0^2|H_1^T\epsilon H_2|^2+k^2|N^\dagger N|^2 \\ &+|N|^2(\tilde{\lambda}_1^2|H_1|^2+\tilde{\lambda}_2^2|H_2|^2) \\ &+k\lambda_0[(H_1^T\epsilon H_2)^\dagger N^2+\text{h. c.}] \\ &+[\tilde{\lambda}_1\mu_1|H_1|^2N+\tilde{\lambda}_2\mu_2|H_2|^2N \\ &+\tilde{\lambda}_0\mu_0(H_1^T\epsilon H_2)^\dagger N+\text{h. c.}] \\ &+[k\mu_0N^\dagger N^2+\text{h. c.}]\ . \end{aligned} \tag{2}$$

The potential has 9 parameters: $\tilde{\lambda}_0$, $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, μ_0, μ_1, μ_2, k, x and $\tan\beta=v_2/v_1$, where $x=<N>_0$ is the vacuum expection value (VEV) of the Higgs singlet N and v_1 res. v_2 are those of H_1 and H_2

$$\begin{aligned} H_1 &= \begin{pmatrix} v_1+\frac{1}{\sqrt{2}}(\eta_1+ia_1) \\ h_1^- \end{pmatrix} \\ H_2 &= \begin{pmatrix} h_2^+ \\ v_2+\frac{1}{\sqrt{2}}(\eta_2+ia_2) \end{pmatrix} \\ N &= x+\frac{1}{\sqrt{2}}(\eta_N+ia_N) \end{aligned} \tag{3}$$

$\partial V/\partial\eta_i=0$ yields three extremum conditions [6] which can be cast into the following relations (assuming parameters to be real).

$$\begin{aligned} \left(\mu_1+\tilde{\lambda}_1x\right) &= \left(2\mu^2\sin^2\beta-\frac{1}{2}m_Z^2\cos 2\beta\right)^{\frac{1}{2}} \\ \left(\mu_2+\tilde{\lambda}_2x\right) &= \left(2\mu^2\cos^2\beta+\frac{1}{2}m_Z^2\cos 2\beta\right)^{\frac{1}{2}} \\ \mu_0+2kx &= \frac{\tilde{\lambda}_0v^2}{4\mu^2}\Big[\tilde{\lambda}_1\left(\mu_1+\tilde{\lambda}_1x\right)\cot\beta \\ &\quad+\tilde{\lambda}_2\left(\mu_2+\tilde{\lambda}_2x\right)\tan\beta\Big] \end{aligned} \tag{4}$$

with

$$\mu^2:=-\frac{1}{4}\tilde{\lambda}_0\left(\tilde{\lambda}_0v^2+\frac{2\mu_0x+2kx^2}{\cos\beta\sin\beta}\right)\ . \tag{5}$$

With these relations three parameters can be eliminated. One obtains for μ^2

$$\mu^2=\frac{1}{2}\left(\mu_1+\tilde{\lambda}_1x\right)^2+\frac{1}{2}\left(\mu_2+\tilde{\lambda}_2x\right)^2 \tag{6}$$

and

$$\mu_{\min}=\begin{cases} \frac{1}{2}m_Z\sqrt{\tan^2\beta-1} \text{ for } \tan\beta\geq 1 \\ \frac{1}{2}m_Z\sqrt{\cot^2\beta-1} \text{ for } \tan\beta<1 \end{cases} \tag{7}$$

The quartic coupling constants of the Higgs doublets are $(g_1^2+g_2^2)/8$, $g_2^2/4$ and $\tilde{\lambda}_0$. And these coupling constants determine the tree level upper bound of m_{S_1}, the mass of the lightest Higgs scalar. It is given by

$$\begin{aligned} m_{S_1}^2 &\leq \left(\frac{g_1^2+g_2^2}{2}\cos^2\beta+\tilde{\lambda}_0^2\sin^2 2\beta\right)(v_1^2+v_2^2) \\ &= m_Z^2\left(\cos^2 2\beta+2\frac{\tilde{\lambda}_0^2}{(g_1^2+g_2^2)}\sin^2 2\beta\right) \end{aligned} \tag{8}$$

3 Radiative corrections and renormalization group evolution

The potential given by (2) holds on tree-level, neglecting radiative corrections. In order to calculate these, we first note, that supersymmetry obviously implies some nontrivial relations between the couplings in the Higgs-potential. E.g. we have the gauge couplings in them and some relations between the trilinear terms in the potential and the bilinear and quartic terms. Above the scale of SUSY breaking, Ward identies between the vertex functions assure that these relations are maintained even after radiative corrections are taken into account. Below the scale of SUSY breaking, radiative corrections will spoil these relations and one has to consider a more general Higgs-potential. Since in the nonlinear model supersymmetry is broken at the Planck-scale, the parameters in the potential should be considered as parameters renormalized at this scale. Below that scale a more general potential must be used and the parameters can be obtained by solving the renormalization group equations (RGE) of this potential. In order to find this potential, one notes that the interactions involving dimensionless couplings have a chiral Peccei-Quinn (PQ-) symmetry

$$\begin{aligned} H_1\to e^{i\alpha}H_1 \quad H_2\to e^{i\alpha}H_2 \quad N\to e^{-i\alpha}N \\ f_L\to e^{i\alpha/2}f_L \quad f_R\to e^{-i\alpha/2}f_R \end{aligned} \tag{9}$$

where $f_{L,R}$ are the left and right handed fermion fields coupling to the Higgs-doubletts. This symmetry is broken only softly be mass terms which we assume to be of the order M_Z. We also assume that all parameters of the potential are real, so that CP is not explicitly broken. With this in mind, we are ready to write down the most general gauge invariant Higgs-potential involving two doublets and a singlett with softly broken PQ-symmetry. It reads

$$\begin{aligned} V = &\Lambda(N+N^*)+\mu_1^2\Phi_1^\dagger\Phi_1+\mu_2^2\Phi_2^\dagger\Phi_2 \\ &+\mu_3^2N^*N+\mu_4^2\left(\Phi_1^\dagger\Phi_2+\text{h.c.}\right) \\ &+\mu_5^2(N^2+\text{h.c.}) \\ &+f_1\Phi_1^\dagger\Phi_1\,(N+N^*)+f_2\Phi_2^\dagger\Phi_2\,(N+N^*) \\ &+f_3\left(\Phi_1^\dagger\Phi_2N+\text{h.c.}\right)+f_4\,(N^*NN+\text{h.c.}) \\ &+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2+\frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\ &+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \end{aligned}$$

$$+\frac{k_1}{2}(\Phi_1^\dagger\Phi_1)(N^*N) + \frac{k_2}{2}(\Phi_2^\dagger\Phi_2)(N^*N) + \frac{k_3}{2}\left((\Phi_1^\dagger\Phi_2)N^2 + \text{h.c.}\right) + \frac{k_4}{2}(N^*N)^2 \qquad (10)$$

where we used the notation $\Phi_1 = H_1$ and $\Phi_2 = \epsilon H_2^*$. At the Planck scale (10) has to match (2) which yields the following boundary conditions for solving the RGE

$$\begin{aligned}
\Lambda(M_P) &= 0\\
\mu_3^2(M_P) &= \mu_0^2(M_P)\\
\mu_4^2(M_P) &= 0\\
\mu_5^2(M_P) &= 0\\
f_1(M_P) &= \tilde\lambda_1(M_P)\mu_1(M_P)\\
f_2(M_P) &= \tilde\lambda_2(M_P)\mu_2(M_P)\\
f_3(M_P) &= \tilde\lambda_0(M_P)\mu_0(M_P)\\
f_4(M_P) &= k(M_P)\mu_0(M_P)\\
\lambda_1(M_P) &= \frac{g_1(M_P)^2 + g_2(M_P)^2}{4}\\
\lambda_2(M_P) &= \frac{g_1(M_P)^2 + g_2(M_P)^2}{4}\\
\lambda_3(M_P) &= \frac{g_2(M_P)^2 - g_1(M_P)^2}{4}\\
\lambda_4(M_P) &= \lambda_0(M_P)^2 - \frac{1}{2}g_2(M_P)^2\\
k_1(M_P) &= 2\tilde\lambda_1^2(M_P)\\
k_2(M_P) &= 2\tilde\lambda_2^2(M_P)\\
k_3(M_P) &= 2k\tilde\lambda_0(M_P)\\
k_4(M_P) &= 2k^2(M_P) \qquad (11)
\end{aligned}$$

Below M_P the parameters in (11) will evolve as solutions of the RG equations:

$$\begin{aligned}
\frac{d\Lambda}{dt} &= \frac{1}{32\pi^2}\left(8\mu_1^2 f_1 + 8\mu_2^2 f_2 + 8\mu_3^2 f_4 + 8\mu_4^2 f_3 + 8\mu_5^2 f_4\right)\\
\frac{d\mu_1^2}{dt} &= \frac{1}{32\pi^2}\left(12\lambda_1\mu_1^2 + (8\lambda_3 + 4\lambda_4)\mu_2^2 \; 2k_1\mu_3^2 + 8f_1^2 + 4f_3^2\right)\\
\frac{d\mu_2^2}{dt} &= \frac{1}{32\pi^2}\left((8\lambda_3 + 4\lambda_4)\mu_1^2 + 12\lambda_2\mu_2^2 + 2k_2\mu_3^2 + 8f_2^2 + 4f_3^2\right)\\
\frac{d\mu_3^2}{dt} &= \frac{1}{32\pi^2}\left(4k_1\mu_1^2 + 4k_2\mu_2^2 + 8k_4\mu_3^2 + 8f_1^2 + 8f_2^2 + 8f_3^2 + 24f_4^2\right)\\
\frac{d\mu_4^2}{dt} &= \frac{1}{32\pi^2}\left((4\lambda_3 + 8\lambda_4)\mu_4^2 + 4k_3\mu_5^2 \; 4f_1f_3 + 4f_2f_3\right)\\
\frac{d\mu_5^2}{dt} &= \frac{1}{32\pi^2}\left(4k_4^2\mu_5^2 + 4k_3^2\mu_4^2 + 4f_2^2 + 4f_1^2 + 8f_4^2\right)\\
\frac{df_1}{dt} &= \frac{1}{32\pi^2}\,(12\lambda_1 + 4k_1)f_1 + (8\lambda_3 + 4\lambda_4)f_2 + 4k_3f_3 + 4k_1f_4)\\
\frac{df_2}{dt} &= \frac{1}{32\pi^2}\,(8\lambda_3 + 4\lambda_4)f_1 + (12\lambda_1 + 4k_1)f_2 + 4k_3f_3 + 4k_2f_4)\\
\frac{df_3}{dt} &= \frac{1}{32\pi^2}\,(4k_3f_1 + 4k_3f_2 + (4\lambda_3 + 8\lambda_4 + 2k_1 + 2k_2)f_3 + 4k_3f_4)\\
\frac{df_4}{dt} &= \frac{1}{32\pi^2}\,(4k_1f_1 + 4k_2f_2 + 4k_3f_3 + 20k_4f_4)\\
\frac{d\lambda_1}{dt} &= \frac{1}{16\pi^2}\left(12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + \frac{1}{2}k_1^2 - \lambda_1(3g_1^2 + 9g_2^2 - 12h_b^2) + \frac{3}{2}g_2^4 + \frac{3}{4}(g_1^2 + g_2^2)^2 - 12h_b^4\right)\\
\frac{d\lambda_2}{dt} &= \frac{1}{16\pi^2}\left(12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + \frac{1}{2}k_2^2 - \lambda_2(3g_1^2 + 9g_2^2 - 12h_t^2) + \frac{3}{2}g_2^4 + \frac{3}{4}(g_1^2 + g_2^2)^2 - 12h_t^4\right)\\
\frac{d\lambda_3}{dt} &= \frac{1}{16\pi^2}\left(4\lambda_3^2 + 2\lambda_4^2 + (\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + \frac{1}{2}k_1k_2 + \lambda_3(3g_1^2 + 9g_2^2 - 6h_b^2 - 6h_t^2) + \frac{9}{4}g_2^4 + \frac{3}{4}g_1^4 - 12h_b^2h_t^2\right)\\
\frac{d\lambda_4}{dt} &= \frac{1}{16\pi^2}\left(8\lambda_3\lambda_4 + 2\lambda_4(\lambda_1 + \lambda_2) + 4\lambda_4^2 + 2k_3^2 + \lambda_4(3g_1^2 + 9g_2^2 - 6h_b^2 - 6h_t^2) + 12h_b^2h_t^2 + 3g_1^2g_2^2\right)\\
\frac{dk_1}{dt} &= \frac{1}{16\pi^2}\,(6\lambda_1k_1 + 2\lambda_4k_2 + 4\lambda_3k_2 + 4k_1k_4 + 2k_1^2 + 4k_3^2)\\
\frac{dk_2}{dt} &= \frac{1}{16\pi^2}\,(6\lambda_2k_2 + 2\lambda_4k_1 + 4\lambda_3k_1 + 4k_2k_4 + 2k_2^2 + 4k_3^2)\\
\frac{dk_3}{dt} &= \frac{1}{16\pi^2}\,(2k_1k_3 + 2k_2k_3 + 2k_3k_4 + 2\lambda_3k_3 + 4\lambda_4k_3)\\
\frac{dk_4}{dt} &= \frac{1}{16\pi^2}\left(k_1^2 + k_2^2 + 2k_3^2 + 10k_4^2\right) \qquad (12)
\end{aligned}$$

These equations were derived from the μ-dependent part of the one loop radiative corrections to the effective potential

$$V_{\text{eff}} = V_0 + \frac{1}{64\pi^2}\text{Str}(M^2(\phi))^2\left(\ln\frac{M^2(\phi)}{\mu^2} - \frac{3}{2}\right) \qquad (13)$$

and the requirement that the full potential does not depend on $t = \ln\mu$.

$$\frac{dV_{\text{eff}}}{dt} = \frac{\partial V_0}{\partial X_i}\frac{dX_i}{dt} - \frac{1}{32\pi^2}\text{Str}(M^2(\phi))^2 = 0 \qquad (14)$$

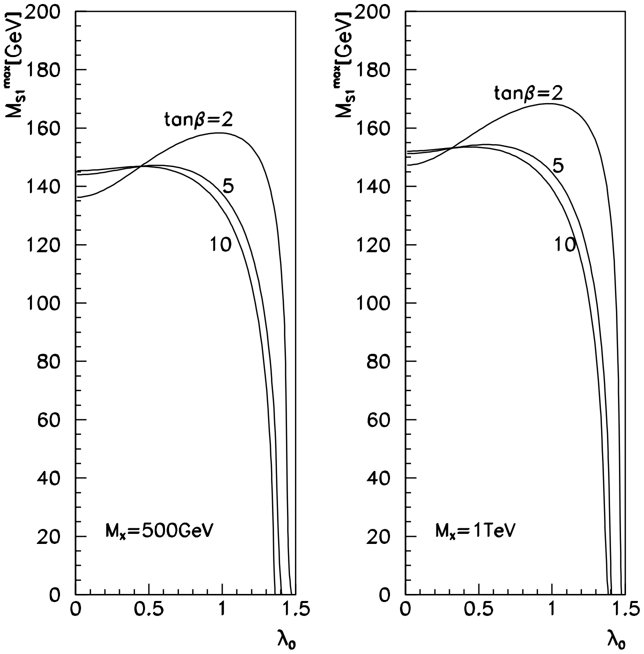

Fig. 1. Upper bound on the mass of the lightest scalar as function of λ_0. The topmass is 175 GeV

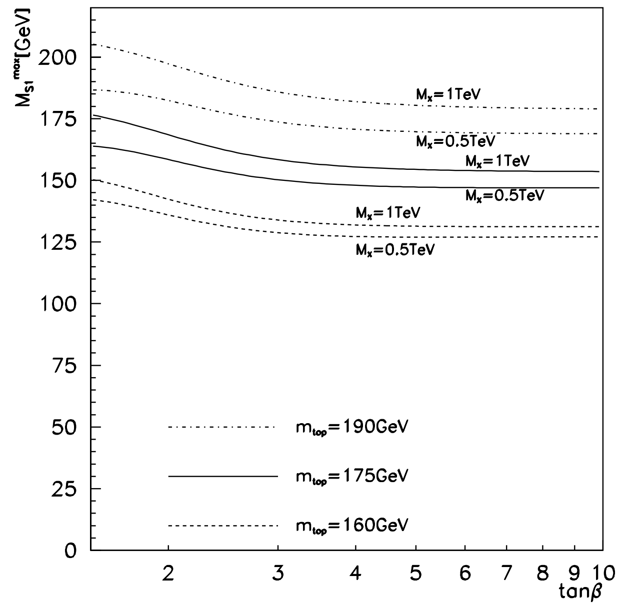

Fig. 2. Upper bound on the mass of the lightest scalar as function of $\tan\beta$

Here ϕ represents collectively all scalar fields and X_i all parameters of the potential. $M^2(\phi)$ is the field dependent mass matrix of the model. Str is the supertrace operator. When calculating the supertrace, fermions enter with a negative sign, bosons with a positive sign and all modes have to be weighted according to their spin- and colour-degenaracy. From the potential (10) one derives the tree level mass matrix

$$\mathcal{M}_S^2 = \begin{pmatrix} 2\lambda_1 v^2\cos^2\beta + \Sigma_1\tan\beta & -\Sigma_1 + (\lambda_3+\lambda_4)v^2\sin 2\beta & -\Sigma_2\sin\beta + 2(k_1 xv + 2f_1 v)\cos\beta \\ -\Sigma_1 + (\lambda_3+\lambda_4)v^2\sin 2\beta & 2\lambda_2 v^2\sin^2\beta + \Sigma_1\cot\beta & \Sigma_2\cos\beta + 2(k_2 xv + 2f_2 v)\sin\beta \\ \Sigma_2\sin\beta + 2(k_1 xv + 2f_1 v)\cos\beta & \Sigma_2\cos\beta + 2(k_2 xv + 2f_2 v)\sin\beta & -\frac{\Sigma_3}{x} + 3f_4 x + 2k_4 x^2 \end{pmatrix} \quad (15)$$

with $\Sigma_1 = -(\mu_4^2 + f_3 x + k_3 x^2)$, $\Sigma_2 = f_3 v + 2k_3 xv$ and $\Sigma_3 = \Lambda + f_2 v^2\sin^2\beta + f_1 v^2\cos^2\beta + \frac{f_3}{2}v^2\sin 2\beta$. $\mu_1 \ldots \mu_3$ have been eliminated in favor of the vaccuum expectation values of the Higgs-fields using the tree-level minimum conditions at the electroweak scale.

At this scale there still will be some radiative corrections which can be handled in the effective potential formalism. After one has calculated the effective potential (13), the VEVs are given by its minimum and the mass matrix by the matrix of its second derivatives.

4 Upper bound for the mass of the lightest scalar

Similar to the linear supersymmetric model (MSSM or NMSSM) it is possible to derive an upper bound for the mass of the lightest scalar from the potential (10). Performing a orthogonal transformation of tree level matrix (15) like $\mathcal{M}' = U^{\mathrm{tr}}\mathcal{M}U$ with

$$U = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (16)$$

the 11-element of $\mathcal{M}'$ yields an upper bound on the mass of the lightest scalar, since the smallest eigenvalue of a positive definite matrix is smaller than its smallest diagonal elements:

$$M_{S_1}^2 \le (2\lambda_1\cos^4\beta + 2\lambda_2\sin^4\beta + (\lambda_3+\lambda_4)\sin^2 2\beta)v^2 \quad (17)$$

This formula generalizes the bound obtained previously without taking the renormalization group evolution into account. This bound only involves the dimensionless quartic couplings of the potential from which one concludes, that M_{S_1} is still close to the electroweak scale, while the masses of the other parameters involve the remaining mass parameters of the potential and the singlett VEV. As in the linear supersymmetric model these can give rise to scalars masses which much larger than M_Z.

(17) holds on tree level and there will be some corrections which can be calcuted from the effective potential (13). The most dominant contributions will be from the top-quark due to its large Yukawa-coupling but also the scalar selfinteractions can give rise to significant contributions. The latter one depend on the details of the bi- and

trilinear terms in the potential. A general discussion can be based on the observation that the mass of the lightest scalar approaches its upper limit if the heavier doublett scalar masses are much larger than $M_Z \approx M_{S_1}$. For discussing an upper bound on M_{S_1}, it is thus sufficient to consider this limiting case, where all heavier scalars have common mass $M_X \gg M_Z$. In this limit, the field $\Phi = \cos\beta\Phi_1 + \sin\beta\Phi_2$ containes only mass eigenstates, namely the lightest scalar and the three Goldstone modes. Below M_X, the heavier scalars decouple and one recovers the Standard model with one Higgs-doublett given by Φ. Its quartic self-interaction parameter λ is then fixed by

$$\lambda(M_X) = \lambda_1(M_X)\cos^4\beta + \lambda_2(M_X)\sin^4\beta + \frac{1}{2}(\lambda_3(M_X) + \lambda_4(M_X))\sin^2 2\beta \qquad (18)$$

One can then use the Standard model RGE in order to calculate the coupling and thus the Higgs-mass at some lower scale. Corrections to the tree level formula

$$M_h^2 = 2\lambda v^2 \qquad (19)$$

remain small, if λ is renormalized at the top-mass, since the potentially large contributions from the top-Yukawa-interaction vanish with this choice. The bound obtained by this procedure then depends on the following input parameters: the topmass, the vacuum angle $\tan\beta$, the mass scale of the heavier scalars M_X and the free dimensionless parameters at M_P, namely $\tilde\lambda_0, \tilde\lambda_1, \tilde\lambda_2$ and k. To be definite, we summarize the numerical procedure to obtain the upper bound for M_{S_1}.

1. For invoking the boundary conditions for the integration of the RGE at M_P, we need the values of gauge and Yukawa-couplings at this scale. So we first start with their known values at M_Z. For the gauge couplings in the $\overline{MS}$-renormalization scheme, these read $g_1^2(M_Z) = 0.1279$, $g_2^2(M_Z) = 0.4239$ and $g_3 = 1.458$ [7].
2. For the evolution of the gauge couplings we use their two-loop β-function[8]. From M_Z to m_t we have 5 active quark flavours and one Higgs-doublet. At m_t, the Yukawa-coupling in the $\overline{MS}$-scheme is obtained from $m_t^{\mathrm{pole}} = h_t(m_t)v(m_t)(1 + \frac{3}{4\pi}\alpha_s(m_t))$[9].
3. We then use the standard model RGE with 6 flavors to evolve further to M_X, where the heavy scalars are activated. The Yukawa-couplings get rescaled at this point according to $h_t \to h_t/\sin\beta$ and $h_b \to h_b/\cos\beta$.
4. Now evolve to M_P using the β-functions of the full model. At one loop level, the, up to now unknown, parameters of the Higgs potential do not enter into the β-functions of the gauge- and Yukawa-couplings, while for the two loop part only the h_t and α_s-contributions are significant.
5. At M_P, choose the remaining input parameters $\tilde\lambda_0$, $\tilde\lambda_1$, $\tilde\lambda_2$ and k and use (11) in order to fix the quartic couplings of the Higgs potential.
6. Now run steps 2. to 4. in reversed order, to get the scalar self interaction at m_t, from which $M_{S_1}^{\max}$ can be obtained as discussed above.

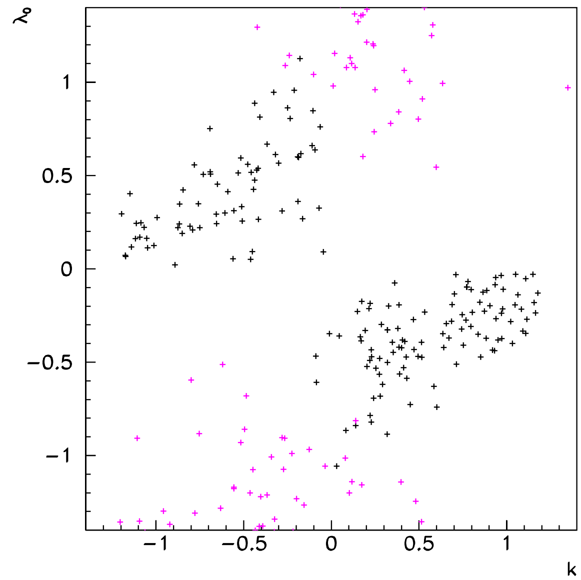

Fig. 3. Resulting scatter plots of points in the $\lambda_0 - k$-plane compatible with electroweak symmetry (black points). 50000 random points out of the parameter range (20) where used as input. The bright points correspond to allowed parameter-sets, when one neglects the renormalization group evolution from M_P to the electroweak scale

This upper bound is a function of m_t, $\tan\beta$, M_X, $\tilde\lambda_0$, $\tilde\lambda_1$, $\tilde\lambda_2$ and k. It turns out, that $M_{S_1}^{\max}$ is a monotonically decreasing function of $\tilde\lambda_1$, $\tilde\lambda_2$ and k. This can be understood from the fact that nonvanishing values of these couplings add positive contributions to the β-functions of λ_1 and λ_2. So these couplings are driven to smaller values if one moves down from M_P to M_Z. For sufficiently large values of $\tilde\lambda_1$ and $\tilde\lambda_2$, $m_{S_1}^2$ will be become negative which means that the Higgs-potential becomes instable at the electroweak scale. This implies an upper bound of $|\tilde\lambda_1| \lesssim 0.8$ and $|\tilde\lambda_2| \lesssim 1.2$ for the couplings at the Planck-scale. Nonvanashing values of k however don't produce instabilities for $|k| \lesssim 3.0$. In order to find an absolute upper bound on $M_{S_1}^2$, we set $\tilde\lambda_1 = \tilde\lambda_2 = k = 0$. Figure 1 plots $M_{S_1}^{\max}$ as a function of $\tilde\lambda_0$ for different values of $\tan\beta$. For small values of $\tan\beta$ the bound first rises as $\tilde\lambda_0$ increases and then drops down to 0, implying an upper bound on $\tilde\lambda_0$ between 1.2 and 1.4. Figure 2 shows an absolute upper bound on $M_{S_1}^{\max}$ as a function of $\tan\beta$ for different values of m_t and M_X. Here $\tilde\lambda_0$ has been varied within the allowed domain and the largest possible value of $M_{S_1}^{\max}$ has been used as an absolute upper bound.

We note that in evalutating these upper bounds all parameters were treated as independend. However, there are some additional constraints from the requirement that electroweak symmetry is broken at a scale $v = 176$GeV. These constraints may result in a situation that the absolute upper bound plotted in Fig. 1 is not actually reached in the physically allowed region of the parameter space.

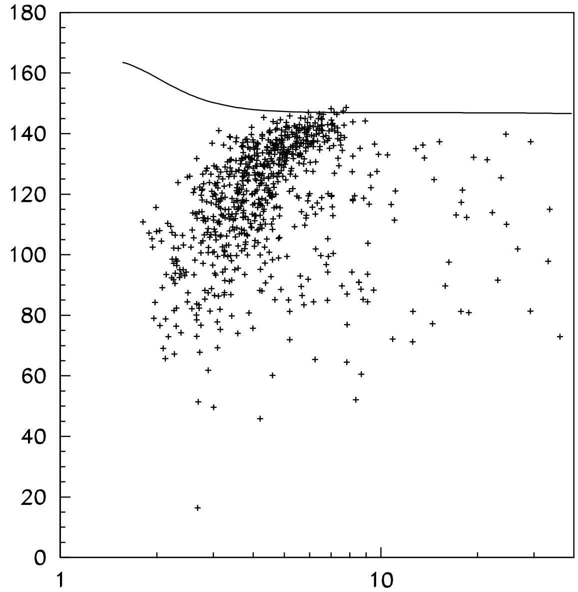

Fig. 4. Allowed domain in the M_{S_1}-tan β-plane. The full line corresponds to the absolute upper bound from Fig. 2, evaluated with $m_t = 175$GeV and $M_X = 500$GeV

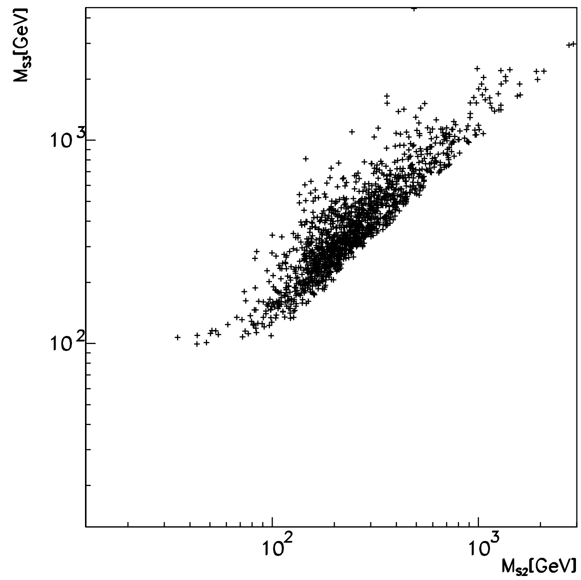

Fig. 5. Masses of the heavier scalars M_{S_2} and M_{S_3}

5 Electroweak symmetry breaking

For the calculation of the upper bound of the lightest Higgs mass only the evolution of the dimensionless couplings were investigated and correct electroweak symmetry breaking was assumed. Since by construction of the potential the μ_i^2 are all positive, the question arises, whether electroweak symmetry breaking occurs at all and if the requirement of electroweak symmetry breaking introduces some further constraints on the allowed parameter-range.

In order to investigate this question, we start by selecting random input parameters at M_P:

$$\begin{aligned} -1.2 < \tilde{\lambda}_i &< 1.2 \quad i = 0,1,2 \\ -1.2 < \ k \ &< 1.2 \\ 0 < \mu_i^2 &< 1\text{TeV} \quad i = 0,1,2 \\ 0.92 < h_t & \ 1.1 \end{aligned} \tag{20}$$

The choice of this parameter range for the λ_i is motivated by the discussion of vacuum stability at the electroweak scale in the previous section. We then use the RGE (12) to calculate the parameters at the electroweak scale. At the electroweak scale we calculate the derivatives of the effective potential and try to find a non-trivial solution of the minimum equations. If a minimum is found, its scale $v' = \sqrt{\langle H_1 \rangle_0^2 + \langle H_2 \rangle_0^2}$ usually has not yet the correct value of 176GeV. We then iteratively rescale the input masses $\mu_i \to 176\text{GeV}/v'$ and repeat the integration of the RGE and the minimization procedure until the correct scale of symmetry breaking is obtained.

The condition of electroweak symmetry breaking establishes some constraints on the allowed parameter range of the model. Figure 3 shows the allowed domain in $\lambda_0 - k$-plane and compares it with the results obtained, when the renormalization group evolution and radiative corrections are neglected. One notes a significant influence of the renormalization group evolution, especially the sign of the product $\lambda_0 k$ flips.

Finally we compute the mass spectrum from the second derivatives of the effective potential. The resulting masses for the lightest scalar are shown in Fig. 4 as a function of tan β. For comparision, the upper bound calculated previously is also displayed. For tan $\beta \gtrsim 5$ this upper bound is reached by the calculated samples of the parameter range. For small tan β the previously calculated upper bound increases due to the $v_1 v_2$-term in (17). This increase is obviously not observed when one takes the requirement of electroweak symmetry breaking into account. Instead for small tan β the observed upper bound decreases in a way similar to the minimal linear supersymmetric model (but with larger values for M_{S_1}. Figure 5 shows the mass-spectrum for the heavier scalars M_{S_2} and M_{S_3}. For these masses the preferred values are between 200 and 500 GeV. There is no strict upper limit for these masses, but the decreasing density of points in the plot shows, that an increasing amount of fine tuning in the Higgs potential is needed in order to achieve larger masses of the heavier scalars.

6 Conclusion

We have derived the RG equations of the nonlinear supersymmetric standard model on one loop level. Assuming supersymmetry to be broken at the Planck scale, we investigated radiative breaking of electroweak gauge symmetry at the Fermi scale. An upper bound of the lightest Higgs scalar mass was determined to be about 160 GeV.

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